

SHOW ALL YOUR WORK! NO WORK=NO CREDIT!!

Note: Unless otherwise specified, you need not evaluate probabilities to a decimal. For instance, an expression such as $\binom{7}{5}(1/3)^5(2/3)^2$ or $(2/5)e^{-3.4}$ is fine as a final answer.

Note: If you use random variables that are not already defined by the problem, you should define them yourself! And most likely you'll have to state their distribution as well.

1. Suppose that each week you buy a lottery ticket that gives you a chance of $1/10$ of winning a prize.

a) Find the chance that it takes more than 5 weeks before you win a prize.

b) Find the expected number of weeks until you win a prize.

c) Find the expected number of prizes you win in the first 20 weeks.

d) Find the chance that you win your third prize in the 20th week.

2. The half-life of a certain radioactive isotope is 1000 years. If a particular atom of this isotope has already survived 1000 years, what is the chance that it will disintegrate in the next 500 years? Calculate to four decimal places.

(Hint: Model the atom's lifetime with an exponential distribution.)

3. Let X and Y be independent random variables with $E(X) = E(Y) = \mu$, and $\text{Var}(X) = \text{Var}(Y) = \sigma^2$. Calculate $\text{Var}(XY)$.

4. Let X be a random variable with density

$$f(x) = \begin{cases} cx(1-x)^2, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find c .

b) Find $P(X \leq 1/2)$.

c) What is $P(X = 1/3)$?

d) Find $E(X)$.

e) Find $\text{Var}(X)$.

5. A hotel elevator has a capacity of 5000 lbs. Guests at the hotel weigh 150 lbs. on average, with a standard deviation of 55 lbs. Suppose 30 of the hotel's guests get into the elevator. What is the chance that the elevator will be overloaded? Estimate to four decimal places. State what assumption(s) you are making.

6. On average, one cubic inch of Granma's cookie dough contains 2 brown chocolate chips and 1 white chocolate chip. Granma uses exactly two cubic inches of her dough per cookie. I buy one cookie from Granma.

a) What is the chance that this cookie contains at most 2 brown chocolate chips? State your assumptions.

b) What is the chance that the total number of chocolate chips (regardless of color) in the cookie is no more than 5? State your assumptions, if different from those in part (a).

c) Given that the cookie contains 5 chocolate chips total, what is the chance that it contains exactly 2 white and 3 brown chocolate chips? State any new assumptions, and simplify your answer.

7. Let A and B be events with $P(A) = 1/3$ and $P(B) = 1/2$. Let N denote the number of events (A and/or B) that occur.

a) Write N as a sum of indicators.

b) Find $E(N)$.

c) Assume A and B are **independent**. Find $Var(N)$.

d) Assume instead that $A \subset B$. Find $Var(N)$.

8. Two players, A and B , play a sequence of games. Each game is won by A with probability p_A , won by B with probability p_B , or drawn with probability $p_D = 1 - p_A - p_B$. The first player to win a game is declared the overall winner.

a) Calculate the probability of the event

$$A_n := \{A \text{ wins overall, and this takes exactly } n \text{ games}\}, \quad n = 1, 2, \dots$$

b) Show by a rigorous calculation that

$$P(A \text{ wins overall}) = \frac{p_A}{p_A + p_B}.$$

9. Suppose the time you have to wait for the shuttle to Discovery Park has an exponential distribution with mean 10 minutes. If you have already waited 10 minutes, what is the chance the bus will come in the next 2 minutes?

10. **Extra credit!!**

Suppose the number of misprints in a text has a Poisson(μ) distribution. A proof-reader catches each mistake with probability p , independently of the other mistakes. Find the distribution of the number of mistakes found by the proofreader in the whole text.