Math 4610 - Review for Exam 1 (Spring 2016)
SHOW ALL YOUR WORK! NO WORK=NO CREDIT!!

1. A fair 6-sided die is rolled twice.

   a) Write down an outcome space (Ω) for this experiment.

   Define the events:

   $$A = \{\text{the difference between the rolls is 2}\}$$
   $$B = \{\text{both rolls are even}\}$$

   b) List all elements of A, and find $P(A)$.

   c) List all elements of B, and find $P(B)$.

   d) Find $P(A|B)$.

2. Let A, B and C be independent events, with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$, respectively.

   a) Find the probability that exactly one of the events occurs.

   b) Find the probability that at least one of the events occurs.

   c) Find the probability that A occurs, given that exactly one of the events occurs.

3. A fair coin is tossed repeatedly until two heads in a row appear. Calculate the probability P_r that exactly r tosses are needed, for $r = 1, 2, \ldots, 5$.
   (Hint: count the number of "favorable" sequences of heads and tails in r tosses.)

4. Prove that if $B_1, \ldots, B_n$ is a partition of B, then

   $$P(A|B) = P(A|B_1)P(B_1|B) + \cdots + P(A|B_n)P(B_n|B).$$

   (Hint: use the definition of conditional probability!)

5. A hat contains f fair 6-sided dice, and b biased dice that have an extra "1" in place of the "6". A die is drawn at random from the hat and tossed. Given that the outcome of the toss is a "1", what is the probability the die is fair?

6. Consider two boxes: Box 1 contains 1 white ball and 2 black balls, and Box 2 contains 3 white balls and 2 black balls. A box is chosen at random, and a ball is drawn from that box at random.

   a) Find the probability that the ball is white.

   b) Find the probability that Box 1 was chosen, given that the ball is white.

   c) Find the probability that Box 1 was chosen, given that the ball is black.

Suppose that after seeing the ball, you must guess which box it came from. Your strategy is to guess the box with the highest posterior probability given the color of the ball.

d) Draw a suitable tree diagram, and label each arrow with the appropriate probability or conditional probability.

e) What is your (unconditional!) probability of guessing correctly?

7. A fair die is rolled seven times.

a) Find the probability that 6 appears at most once.

b) Find the probability that 6 appears twice in the first four rolls, given that 6 appears three times in the seven rolls.

8. Suppose that on average, 4% of purchasers of airline tickets do not show up for their flight. If there are 250 seats available on a flight and 260 tickets were sold, estimate, to 4 decimal places, the probability that the flight will be overbooked. Show all the details!