Problem 18, §3.4

a) Call the teams A and B, and let $A_i =$ "team A wins game i",
$B_i =$ "team B wins game i". Say $P(A_i) = p$, so $P(B_i) = q = 1-p$.
Then

$$P(G=2) = P(A_1A_2) + P(B_1B_2) = p^2 + q^2$$

$$P(G=4) = P\big((A_1B_2 \cup A_2B_1) \cap (A_3A_4 \cup B_3B_4)\big)$$
$$= 2pq\,(p^2+q^2)$$

$$\vdots$$

$$P(G=2k) = P\big((A_1B_2 \cup A_2B_1) \cap (A_3B_4 \cup B_3A_4) \cap \dots \cap (A_{2k-3}B_{2k-2} \cup B_{2k-3}A_{2k-2}) \cap (A_{2k-1}A_{2k} \cup B_{2k-1}B_{2k})\big)$$
$$= (2pq)^{k-1}(p^2+q^2), \quad k = 1, 2, \dots .$$

Note that an even number of games must be played, so

$$P(G=2k-1) = 0, \quad k = 1, 2, \dots .$$

b) Observe that $G = 2T$, where T is the number of "sets" of two games played until one team wins both games in the set.
Then $T \sim \text{geometric}(p^2+q^2)$, so

$$E(T) = \frac{1}{p^2+q^2}, \quad Var(T) = \frac{2pq}{(p^2+q^2)^2}$$

and hence,

$$E(G) = 2E(T) = \frac{2}{p^2+q^2}$$

c) $$Var(G) = 4\,Var(T) = \frac{8pq}{(p^2+q^2)^2}.$$