Answer Key, Hw. #11

§3.3 (12) a) $P(X \geq 20) = P(X - E(X) \geq 10) \leq P(|X - E(X)| \geq 10) \leq \frac{Var(X)}{10^2} = \frac{25}{100} = \frac{1}{4} = 0.25.$

b) If $X \sim$ binomial (n,p), then $np = E(X) = 10$ and $\sqrt{npq} = SD(X) = 5$. But then $\sqrt{10q} = 5 \Rightarrow 10q = 25 \Rightarrow q = 2.5$: impossible! So X can't be binomial.

§3.3 (17) $E(X) = (-1)\cdot\frac{1}{4} + 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} = \frac{1}{4}$, $E(X^2) = 1\cdot\frac{1}{4} + 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} = \frac{3}{4}$

$\Rightarrow E(S) = 25E(X) = \frac{25}{4} = 6.25$, $Var(S) = 25\,Var(X) = 25\left(\frac{3}{4} - \frac{1}{16}\right) = 25\cdot\frac{11}{16}$,

$SD(S) = 5\sqrt{\frac{11}{16}} \approx 4.15.$

a) $P(S<0) \approx \Phi\left(\frac{-\frac{1}{2} - 6.25}{4.15}\right) = \Phi(-1.63) = 1 - \Phi(1.63) = 1 - .9484 = .0516$

b) $P(S=0) \approx \Phi\left(\frac{\frac{1}{2} - 6.25}{4.15}\right) - \Phi\left(\frac{-\frac{1}{2} - 6.25}{4.15}\right) = \Phi(-1.39) - \Phi(-1.63) = .0307$

c) $P(S>0) = 1 - P(S<0) - P(S=0) \approx .9177.$

(19) Let X be the weight of a typical guest, and S the sum of the weights of the 30 people. Then $E(X) = 150$, $SD(X) = 55$ so $E(S) = 30 \cdot 150 = 4500$ and $SD(S) = \sqrt{30}\cdot 55 = 301.2$

Then

$$P(S > 5000) \approx 1 - \Phi\left(\frac{5000 - 4500}{301.2}\right) = 1 - \Phi(1.66)$$
$$= 1 - .9515 = .0485$$

Note: No continuity correction here, since S is already a continuous random variable (i.e., it does not take on integer values only).

§3.4 (4) a) P(there's an "odd-one-out") = 1 − P(all tosses equal)

$= 1 - P(\{hhh, ttt\}) = 1 - \left(\frac{1}{8} + \frac{1}{8}\right) = \frac{3}{4}.$

b) Let T be the number of games until there is an odd-one-out. Then T is geometric with parameter $p = \frac{3}{4}$. So $P(T=r) = \left(\frac{1}{4}\right)^{r-1}\cdot\frac{3}{4}$ for $r = 1, 2, \ldots$;

c) and $E(T) = \frac{1}{p} = \frac{1}{3/4} = \frac{4}{3}.$

§3.4 (12) a) $P(W_1 = W_2) = \sum_{k=1}^{\infty} P(W_1 = k, W_2 = k) = \sum_{k=1}^{\infty} P(W_1 = k)\, P(W_2 = k)$

$$= \sum_{k=1}^{\infty} p_1 (1-p_1)^{k-1} p_2 (1-p_2)^{k-1} = p_1 p_2 \sum_{k=1}^{\infty} \left[(1-p_1)(1-p_2)\right]^{k-1}$$

$$= \frac{p_1 p_2}{1-(1-p_1)(1-p_2)} = \frac{p_1 p_2}{p_1 + p_2 - p_1 p_2}.$$

§3.3 (26) b. $0 \le Var(|X-\mu|) = E(|X-\mu|^2) - (E|X-\mu|)^2 = E[(X-\mu)^2] - (E|X-\mu|)^2$

$\Rightarrow Var(X) = E[(X-\mu)^2] \ge (E|X-\mu|)^2 \Rightarrow SD(X) \ge E|X-\mu|.$

Equality holds if and only if $Var(|X-\mu|) = 0$, which means $|X-\mu|$ is constant.