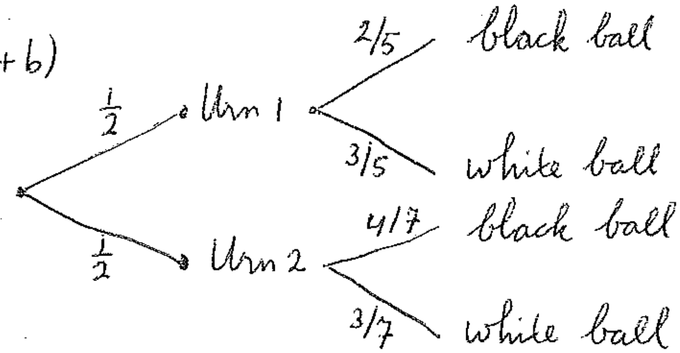
Answer Key, Hw. #3 (Selection)

§1.4 ② $P(\text{not defective and made in city } B) = P(\text{city } B)\, P(\text{not defective} \mid \text{city } B)$

$$= \tfrac{1}{3} \times 0.99 = 0.33, \text{ or } 33\%.$$

⑤ a) + b)



$P(\text{Urn } 1) = P(\text{Urn } 2) = \frac{1}{2}$

$P(\text{black} \mid \text{Urn } 1) = \frac{2}{5}$, $P(\text{black} \mid \text{Urn } 2) = \frac{4}{7}$

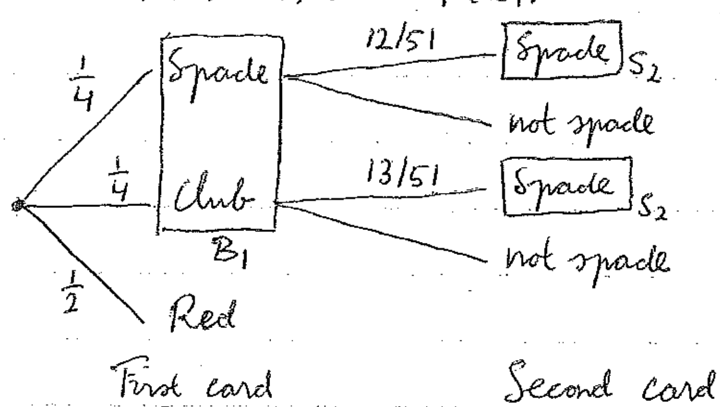
c) $P(\text{black}) = P(\text{black} \mid \text{Urn}1)\, P(\text{Urn}1) + P(\text{black} \mid \text{Urn } 2)\, P(\text{Urn } 2)$

$$= \frac{2}{5} \times \frac{1}{2} + \frac{4}{7} \times \frac{1}{2} = \frac{1}{5} + \frac{2}{7} = \frac{17}{35}.$$

⑥ Let $B_1 = \{\text{first card is black}\}$, $S_i = \{i^{th} \text{ card is spade}\}$, $i = 1, 2$.

Then

$$P(S_2 \mid B_1) = \frac{P(B_1 S_2)}{P(B_1)}.$$



From tree diagram: $P(B_1) = \frac{1}{2}$, $P(B_1 S_2) = \frac{1}{4} \times \frac{12}{51} + \frac{1}{4} \times \frac{13}{51} = \frac{25}{204}$

$$\therefore \quad P(S_2 \mid B_1) = \frac{25/204}{1/2} = \frac{25}{102}.$$

(7) b) Two ways:

(i) If A and B indep., $P(A\cup B) = P(A) + P(B) - P(AB)$

$$= P(A) + P(B) - P(A)P(B) = P(A) + (1-P(A))P(B)$$

$$\Rightarrow 0.8 = 0.5 + 0.5\,P(B) \Rightarrow 0.5\,P(B) = 0.3 \Rightarrow P(B) = 0.6$$

(ii) If A and B indep. then A^c and B^c indep.

$$\Rightarrow 0.8 = P(A\cup B) = 1 - P(A^cB^c) = 1 - P(A^c)P(B^c) = 1 - 0.5\,P(B^c)$$

$$\Rightarrow 0.5\,P(B^c) = 1 - 0.8 = 0.2 \Rightarrow P(B^c) = 0.4 \Rightarrow P(B) = 0.6.$$

(11) Let $I = \{\text{identical twins}\}$, $F = \{\text{fraternal twins}\}$; $P(I) = p$, $P(F) = q = 1-p$.
Let $B_i = \{i^{th} \text{ child is a boy}\}$, $G_i = \{i^{th} \text{ child is a girl}\} = B_i^c$, $i = 1,2$.
Then from the given information:

$$P(B_1B_2|I) = P(G_1G_2|I) = \frac{1}{2}$$

$$P(B_1B_2|F) = P(B_1G_2|F) = P(G_1B_2|F) = P(G_1G_2|F) = \frac{1}{4}.$$

a) $P(\text{both boys}) = P(B_1B_2) = P(B_1B_2|I)P(I) + P(B_1B_2|F)P(F)$

$$= \frac{1}{2}\cdot p + \frac{1}{4}\cdot(1-p) = \frac{2p}{4} + \frac{1-p}{4} = \frac{1+p}{4}.$$

b) $P(\text{first } \cancel{\text{girl}} \text{ boy, second girl}) = P(B_1G_2) = P(B_1G_2|I)P(I) + P(B_1G_2|F)P(F)$

$$= 0\cdot p + \frac{1}{4}\cdot(1-p) = \frac{1-p}{4}.$$

c) $P(B_1) = P(B_1B_2) + P(B_1G_2) = \frac{1+p}{4} + \frac{1-p}{4} = \frac{1}{2}$ (as expected!)

So

$$P(\text{second girl}|\text{first boy}) = P(G_2|B_1) = \frac{P(B_1G_2)}{P(B_1)} = \frac{\left(\frac{1-p}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1-p}{2}.$$

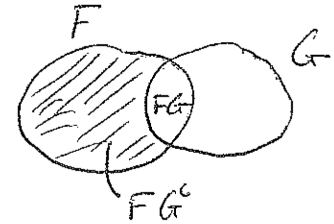
d) $P(G_1) = 1 - P(B_1) = \frac{1}{2}$, and, as in (a), $P(G_1G_2) = \frac{1+p}{4}$.

So

$$P(\text{second girl}|\text{first girl}) = P(G_2|G_1) = \frac{P(G_1G_2)}{P(G_1)} = \frac{\left(\frac{1+p}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1+p}{2}.$$

(12) $P(F \mid G^c) = \dfrac{P(FG^c)}{P(G^c)}$

$= \dfrac{P(F) - P(FG)}{1 - P(G)}$.



§1.5 (5) Let $D = \{\text{person has disease}\}$, $A = \{\text{test is positive}\}$.

a) $P(A) = P(A \mid D)P(D) + P(A \mid D^c)P(D^c) = 0.8 \times 0.01 + 0.05 \times 0.99 = 0.0575$

b) Note that set-theoretically, "but" is equivalent to "and"! Thus,

$P(\text{person has disease but is diagnosed as healthy}) = P(D \cap A^c)$

$= P(A^c \mid D)\,P(D) = [1 - P(A \mid D)]\,P(D) = 0.2 \times 0.01 = 0.002$

c) $P(\text{correctly diagnosed and healthy}) = P(D^c \cap A^c) = P(A^c \mid D^c)\,P(D^c)$

$= 0.95 \times 0.99 = 0.9405$

d) $P(D \mid A) = \dfrac{P(A \mid D)\,P(D)}{P(A)} = \dfrac{0.8 \times 0.01}{0.0575} \approx 0.139$

e) All probabilities that "went in" $[P(D),\ P(A \mid D),\ P(A^c \mid D^c)]$ are proportions of the population, and therefore have a long-run frequency interpretation. Then so do the probabilities that "come out".