Math 2700 - Review for Exam 2 **P. Allaart**

SHOW ALL YOUR WORK! NO WORK=NO CREDIT!!

No Calculators Allowed! - But you shouldn't need any.

1. Compute the following matrix products, if possible.

a) *(7 pts.)* AA^T, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

b) *(5 pts.)* $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 \end{bmatrix} =$

2. *(4 pts. each)* Let A, B and C be $n \times n$ matrices. For each statement below, indicate whether it is always true or not always true. (No explanation needed.)

a) $(AB)C = A(BC)$

b) $(A+B)^{-1} = A^{-1} + B^{-1}$ (Assume all inverses are defined.)

c) $(A+B)(A-B) = A^2 - B^2$

d) $(A+B)^T = A^T + B^T$.

3. *(14 pts.)* Find the inverse of the matrix A. Or, if A is not invertible, explain why not.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 9 & 12 \\ 1 & 5 & 5 \end{bmatrix}$$

4. Let T be the transformation

$$T(x_1, x_2, x_3) = (x_1 + 3x_2 - x_3, -x_1 + x_2 - 3x_3, 2x_1 + 5x_2 - x_3)$$

a) *(5 pts.)* Find the standard matrix A of T.

b) *(12 pts.)* Determine whether T is one-to-one, onto, both, or neither. Show all of the details!

5. a) *(9 pts.)* Find the standard matrix, A, of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which performs a reflection through the line $x_2 = x_1$, followed by a rotation by 90° clockwise about the origin. Explain your method carefully! (A sketch could help.)

b) *(7 pts.)* Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ denote the mapping which reflects points through the line $x_1 = 1$. Explain why S is NOT a linear transformation. (Show by a *specific* example which property of a linear transformation fails.)

6. *(12 pts.)* Suppose the last column of AB is entirely zero but B itself has no column of zeros. What can you say about the columns of A? Explain *as precisely as possible*, including appropriate equations.

7. *(13 pts.)* Suppose A and B are $n \times n$ matrices such that $I + AB$ is invertible. Solve the following system of matrix equations for the *matrices* X and Y:

$$AX + Y = I$$
$$X - BY = I$$

(*Hint*: This is not a routine 2×2 system of linear equations, because the "coefficients" A and B are matrices rather than numbers. You can NOT assume that A and/or B is invertible, so A^{-1} and B^{-1} may not exist! Furthermore, you can NOT divide by a matrix!)

8. **Extra credit!!**

a) *(6 pts.)* Suppose $A(A - 3I) = O$. Does this mean that $A = O$ or $A = 3I$? Give a *proof* or a *counterexample*.

b) *(6 pts.)* Find a natural condition under which $A(A - 3I) = O$ implies that $A = 3I$. Prove that your condition "works".