Math 2700 - Review for Exam 1 **P. Allaart**

1. Describe all solutions of the equation $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where A is row-equivalent to the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

2. Let

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -1 \\ h \\ 4 \end{bmatrix}.$$

For which value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{v_1}$ and $\mathbf{v_2}$?

3. Describe (by an equation and geometrically), the set of all vectors $\mathbf{b}$ in $\mathbf{R}^3$ for which the system is consistent.

$$\begin{aligned} x_1 + 3x_2 \qquad &= b_1 \\ -x_1 - x_2 - x_3 &= b_2 \\ 3x_1 + 7x_2 + x_3 &= b_3 \end{aligned}$$

4. Determine whether the columns of A are linearly independent:

a) $A = \begin{bmatrix} 1 & -1 & 7 \\ 3 & 1 & 13 \\ 2 & 3 & 4 \\ 5 & 7 & 11 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 2 & 1 & 13 & 0 \\ -1 & 5 & -6 & 3 & 8 \\ 0 & 7 & 12 & 3 & 1 \end{bmatrix}$

5. Let

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}.$$

a) Express $\mathbf{b}$ as a linear combination of $\mathbf{v_1}$ and $\mathbf{v_2}$.

b) Suppose A is a matrix such that

$$A \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \mathbf{v_1}, \qquad \text{and} \qquad A \begin{bmatrix} -4 \\ 2 \\ 2 \end{bmatrix} = \mathbf{v_2}.$$

What is the size of A?

c) Let A be the same matrix as in part b). Find a vector $\mathbf{x}$ in $\mathbf{R}^3$ such that

$$A\mathbf{x} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}.$$

Explain which general property you are using. (*Hint:* see part a).)

6. An economy has three sectors: Chemicals, Fuels and Machinery. Chemicals sells 40% of its output to Fuels and 40% to Machinery, and retains the rest. Fuels sells 70% of its output to Chemicals and 30% to Machinery. Machinery sells 50% of its output to Chemicals and 30% to Fuels, and retains the rest.

a) Construct an exchange table for this economy.

b) Set up a system of equations that lead to equilibrium prices at which each sector's income matches its expenses. Write the augmented matrix for the system. **Do not solve the system!**

7. Let A be an $m \times n$ matrix, and let $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be a linearly *dependent* set in $\mathbf{R}^n$. Prove that the set $\{A\mathbf{v_1}, A\mathbf{v_2}, A\mathbf{v_3}\}$ is linearly *dependent.* (Hint: use a dependence relation between $\mathbf{v_1}$, $\mathbf{v_2}$ and $\mathbf{v_3}$.)

8. **Extra credit!!** Suppose A is a 3×3 matrix and $\mathbf{y}$ is a vector in $\mathbf{R}^3$ such that the equation $A\mathbf{x} = \mathbf{y}$ does *not* have a solution. Does there exist a vector $\mathbf{z}$ in $\mathbf{R}^3$ such that the equation $A\mathbf{x} = \mathbf{z}$ has a *unique* solution? Explain!