6/ Find the point on 

\[ x^2 + \frac{y^2}{4} + \frac{z^2}{16} = 3 \]

closest to \((1,1,1)\).

Want to minimize

\[ f(x,y,z) = (x-1)^2 + (y-1)^2 + (z-1)^2 \]

subject to constraint \( g = 3 \), where

\[ g(x,y,z) = x^2 + \frac{y^2}{4} + \frac{z^2}{16} \]

Solve:

\[ \nabla f = \lambda \nabla g \quad \Rightarrow \quad g = 3 \]

Has extremely messy solution:

\( x \approx 1.6213, \ y \approx 1.10595 \)
\( z \approx 1.02454 \)

are the co-ordinates of closest point, which is about \( 0.63 \) \((f \approx 0.397841)\) from \((1,1,1)\).
Optimize \( f(x,y,z) = 2y + z \) subject to the constraints \( g_1 = 12 \) and \( g_2 = 0 \), where
\[
g_1(x,y,z) = x^2 + y^2 + z^2 \quad \text{and} \quad g_2(x,y,z) = y - x^2.
\]

Solve,
\[
\nabla f = 2 \nabla g_1 + \mu \nabla g_2 \Rightarrow g_1 = 12, \quad g_2 = 0
\]

Has 2 solutions
\[
(0,0,-2\sqrt{3}) \quad \text{and} \quad (0,0,2\sqrt{3})
\]

\[
f(0,0,-2\sqrt{3}) = -2\sqrt{3} \quad \text{Min}
\]
\[
f(0,0,2\sqrt{3}) = 2\sqrt{3} \quad \text{Max}
\]