6/ Find the point on

$$x^2 + \frac{y^2}{4} + \frac{z^2}{16} = 3 \quad \text{closest to } (1,1,1).$$

Want to minimize

$f(x,y,z) = (x-1)^2 + (y-1)^2 + (z-1)^2$

subject to constraint $g = 3$, where

$g(x,y,z) = x^2 + \frac{y^2}{4} + \frac{z^2}{16}$

Solve:

$$\nabla f = \lambda \nabla g \quad , \quad g = 3$$

Has <u>extremely</u> messy solution:

$x \approx 1.6213$, $y \approx 1.10595$

$z \approx 1.02454$

are the co-ordinates of closest point, which is about $.63$ ($f \approx .397841$) from $(1,1,1)$.

7/ Optimize $f(x,y,z) = xy + z$ subject to the constraints $g_1 = 12$ and $g_2 = 0$, where $g_1(x,y,z) = x^2 + y^2 + z^2$ and $g_2(x,y,z) = y - x^2$.

Solve.

$$\nabla f = \lambda \nabla g_1 + \mu \nabla g_2 \,;\; g_1 = 12 \,;\; g_2 = 0$$

Has 2 solutions

$$(0, 0, -2\sqrt{3}) \quad \text{and} \quad (0, 0, 2\sqrt{3})$$

$f(0,0,-2\sqrt{3}) = -2\sqrt{3}$ MIN

$f(0,0,2\sqrt{3}) = 2\sqrt{3}$ MAX