Speaker: Idris Assani, Dept of Mathematics, University of North Carolina at Chapel Hill,
Title: On A. Zygmund's differentiation conjecture

Abstract:

The fundamental theorem of calculus provides a differentiation mechanism for continuous real valued functions. A first generalization is given by Lebesgue theorem which states that for a.e. $x \in R$ for each L^1 function f we have

$$\lim_{t \to 0} \frac{1}{2t} \int_{-t}^{t} f(x+u)du = f(x).$$

Consider a bounded measurable vector field v on R^n and a function F defined also on R^n. When can we say that for almost every $x \in R^n$

$$\lim_{t \to 0} \frac{1}{2t} \int_{-t}^{t} F(x+sv(x))ds = F(x)?$$

It is known that the example of the Radon-Nikodym set shows that conditions have to be imposed on the vector field v. A. Zygmund conjectured that the answer is positive if the vector field has unit length, is Lipschitz and the functions $F \in L^2(R^n)$. We will present recent results on this conjecture and discuss three aspects of it: the nature of the vector field, the norm convergence of the averages $\frac{1}{2t}\int_{-t}^{t} F(x+sv(x))ds$ for all L^p functions F, $1 \leq p < \infty$, and the pointwise convergence of the same averages.