

SHOW ALL YOUR WORK! NO WORK=NO CREDIT!!

1. A 13-card hand is dealt from a deck of 52 cards. Find the probability that the hand contains
 - a) exactly two aces
 - b) exactly two aces and three kings
2. Twelve dice are rolled. What is the probability that each number appears exactly twice? [Hint: think about this problem as sampling with replacement.]
3. Let X and Y be independent random variables with $E(X) = E(Y) = \mu$, and $\text{Var}(X) = \text{Var}(Y) = \sigma^2$. Calculate $\text{Var}(XY)$.
4. Two FOUR-sided dice are rolled. Let X_1 and X_2 be the numbers that appear.
 - a) Show the joint distribution table of X_1 and X_2 .
Let $Y_1 = \min(X_1, X_2)$ and $Y_2 = \max(X_1, X_2)$, and define $D := Y_2 - Y_1$.
 - b) Construct the joint distribution table for Y_1 and Y_2
 - c) Give the *range* of D .
 - d) Find the *distribution* of D .
 - e) Find $E(D)$.
5. Suppose the mean family income in a town is \$60,000. Assume family income can not be negative.
 - a) Find a nontrivial upper bound for the percentage of families in this town that make more than \$100,000.
 - b) Find a better bound if it is given furthermore that the standard deviation of family income is \$10,000.
6. There are 3 urns. The first urn contains one red ball and two black balls, the second urn contains three red balls and one black ball, and the third urn contains two balls of each color. A sample of 3 balls is drawn by taking one ball at random from each urn. Let X denote the number of red balls in the sample.
 - a) Does X have a binomial distribution? Explain.
 - b) Find $E(X)$. [Hint: this can be done without calculating the distribution of X !]
7. A company produces electronic devices that work properly with probability 0.9, independently of each other. The devices are sold in boxes of 50 each. Find the largest k such that at least 90% of the boxes have the property that they contain k or more working devices.
8. A seed manufacturer sells seeds in packets of 50. Each seed germinates with a chance of 99%, independently of all others. The manufacturer promises to replace any packet that has 3 or more seeds that do not germinate.

Suppose you buy a single packet. What is the probability that the manufacturer has to replace the packet? Use the Poisson approximation, and estimate to 4 decimal places. (Be careful!)