Math 4610 - Review for Final Exam **P. Allaart**

SHOW ALL YOUR WORK! NO WORK=NO CREDIT!!

1. Two players are each dealt seven cards from a deck of 52. What is the probability that each player gets exactly two aces?

2. A box contains two coins. One is fair, the other lands heads 2/3 of the time. A coin is picked at random from the box and tossed 5 times. If 4 of the tosses are heads, what is the (conditional) probability that the fair coin was chosen?

3. Let X be a random variable with density

$$f(x) = 3x^{-4}, \qquad x \geq 1.$$

Find the mean μ, the median m, and the standard deviation σ of X, and verify that $|\mu - m| \leq \sigma$.

4. Suppose IQ scores in a large population have a mean of 100.

a) Assume IQ scores are nonnegative. Without making any further assumptions about the distribution of the scores, find an upper bound on the percentage of scores exceeding 130.

b) Find a better upper bound for the percentage in part (a), if it also known that the SD is 10.

c) Estimate the percentage in part (a) under the assumption that the distribution of IQ scores is normal with mean 100 and SD 10.

5. Let X and Y be random variables with joint density

$$f(x, y) = 6y, \qquad 0 \leq x \leq 1, \quad 0 \leq y \leq 1 - x.$$

a) Find the marginal densities of X and Y. Specify where the expressions are valid.

b) Find the conditional density of Y given $X = x$. Specify where the expression is valid.

c) Find $E(Y|X = 1/2)$.

6. Let A, B and C be events in some probability space with $P(A) = 1/6$, $P(B) = 1/4$ and $P(C) = 1/3$, and let N be the number of events from among A, B and C that occur. (For instance, if A and B happen but C does not happen, then $N = 2$.)

a) Find $E(N)$. Or is this impossible without more information?

b) Assume A, B and C are independent. Find $\text{Var}(N)$.

c) Assume instead that $A \subset B \subset C$. Find $\text{Var}(N)$.

7. A freight train carries containers for 20 different customers. Each customer independently supplies a random number of containers that is Poisson distributed with parameter $\mu = 4$. Let N be the total number of containers on the train.

a) State the distribution of N.

b) Find $E(N)$ and $\mathrm{Var}(N)$.

c) Use the normal approximation to find the probability that the train carries more than 100 containers.

8. An ambulance station, 30 miles from one end of a 100-mile road and 70 miles from the other end, services accidents along the whole road. Suppose accidents occur with a uniform distribution along the road, and the ambulance can travel at 60 miles per hour. Let T be the time the ambulance needs to get to the accident, in minutes.

a) What is the range of T?

b) Find the c.d.f. of T. (*Hint*: position the ambulance station at the origin, and the road along the x-axis. *Something* has a uniform distribution, but it's not T!)

9. The number of misprints in a document has a Poisson distribution with mean 3. A proofreader finds any given mistake with probability 0.9, independently of the others. Find, to four decimal places, the chance that the proofreader finds at least 2 mistakes.

10. Let (X, Y) be a point chosen at random in the unit disk $x^2 + y^2 \leq 1$, and let $Z = X^2 + Y^2$. Find the density function of Z.

11. In a sequence of independent tosses of a fair coin, let X denote the number of heads in the first 100 tosses, and Y the number of heads in the first 500 tosses. Compute $\mathrm{Corr}(X, Y)$.

12. Let X and Y have joint density

$$f(x,y) = \begin{cases} \lambda^2 e^{-\lambda x}, & 0 < y < x \\ 0, & \text{otherwise,} \end{cases}$$

where $\lambda > 0$. Find $\mathrm{Cov}(X, Y)$.