§3.5 ① Let N be the number of people in the sample who are over 6' 3" tall. Then $N \sim$ binomial (200, 0.01). Since $p = 0.01$ is close to zero, approximate with a Poisson (μ) distribution, where $\mu = np = (200)(0.01) = 2$:

$$P(N \geq 4) = 1 - P(N \leq 3) \approx 1 - \sum_{k=0}^{3} e^{-2} \frac{2^k}{k!} = 0.1428$$

⑥ Assume rain falls steadily and each raindrop falls at a spot independently of other raindrops. Let N be the number of raindrops that fall on the given square inch during the given 10-second interval. It is then reasonable to assume that N is Poisson with parameter $\mu = 30 \times \frac{10}{60} = 5$, and $P(N=0) = e^{-5} = 0.0067$.

⑨ a) $P(X=1, Y=2) = P(X=1)P(Y=2) = e^{-1} \cdot \frac{1^1}{1!} \cdot e^{-2} \cdot \frac{2^2}{2!} = 2e^{-3} = 0.09957$

b) Note that $X+Y$ is Poisson with parameter $1+2=3$. Thus,

$$P\left(\frac{X+Y}{2} \geq 1\right) = P(X+Y \geq 2) = 1 - P(X+Y=0) - P(X+Y=1) = 1 - e^{-3} - 3e^{-3} = 0.8009$$

[Careful! $\frac{X+Y}{2}$ does not have a Poisson distribution!]

c) $$P\left(X=1 \,\middle|\, \frac{X+Y}{2}=2\right) = P(X=1 \mid X+Y=4) = \frac{P(X=1, X+Y=4)}{P(X+Y=4)} = \frac{P(X=1, Y=3)}{P(X+Y=4)}$$

$$= \frac{P(X=1)P(Y=3)}{P(X+Y=4)} = \frac{e^{-1} \cdot e^{-2}\frac{2^3}{3!}}{e^{-3} \cdot \frac{3^4}{4!}} = \frac{8/6}{81/24} = \frac{32}{81} = 0.3951.$$

§3.5: ⑩ a) $E(3X+5) = 3E(X) + 5 = 3\lambda + 5$

b) $\text{Var}(3X+5) = 9\,\text{Var}(X) = 9\lambda$.

c) $$E\left(\frac{1}{1+X}\right) = \sum_{k=0}^{\infty} \frac{1}{1+k} P(X=k) = \sum_{k=0}^{\infty} \frac{1}{1+k} \cdot e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

$$= e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{(k+1)!} = \lambda^{-1} e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{(k+1)!} = \lambda^{-1} e^{-\lambda} \sum_{j=1}^{\infty} \frac{\lambda^j}{j!} \quad (j = k+1)$$

$$= \lambda^{-1} e^{-\lambda} \left(\sum_{j=0}^{\infty} \frac{\lambda^j}{j!} - 1\right) = \lambda^{-1} e^{-\lambda}(e^{\lambda} - 1) = \lambda^{-1}(1 - e^{-\lambda}) = \boxed{\frac{1 - e^{-\lambda}}{\lambda}}$$