§3.2 (2) Let $x_1, \dots, x_{100}$ be the first list, $y_1, \dots, y_{100}$ the second list, and $z_1, \dots, z_{100}$ the third.

a) $\bar{z} = (z_1 + \dots + z_{100})/100 = (x_1+y_1+ \dots + x_{100}+y_{100})/100 = (x_1 + \dots + x_{100})/100 + (y_1 + \dots + y_{100})/100$

$= \bar{x} + \bar{y} = (0.2 \times 1 + 0.8 \times 2) + (0.5 \times 3 + 0.5 \times 5) = 1.8 + 4 = 5.8$

(Note: no assumptions were made regarding the order of ones, twos, threes and fives in the lists!)

b) Similarly, if $z_i = x_i - y_i$ for $i = 1, \dots, n$, then $\bar{z} = \bar{x} - \bar{y} = 1.8 - 4 = -2.2$

c) & d) If $z_i = x_i y_i$ or $z_i = x_i / y_i$ for $i = 1, \dots, n$, then $\bar{z}$ depends on the order within the lists! e.g. for $z_i = x_i y_i$,

Case 1.
$$\begin{aligned}
x_i &: 1, 1, \dots, 1, 2, 2, \dots\dots 2 \\
y_i &: 3, 3, \dots 3, 3, \dots, 3, 5, 5, \dots, 5 \\
z_i &: \underbrace{3, 3, \dots, 3}_{20}, \underbrace{6, \dots, 6}_{30}, \underbrace{10, \dots, 10}_{50} \Rightarrow \bar{z} = 7.4
\end{aligned}$$

Case 2.
$$\begin{aligned}
x_i &: 2, 2, \dots\dots, 2, 1, 1, \dots, 1 \\
y_i &: 3, 3, \dots, 3, 5, \dots, 5, 5, \dots, 5 \\
z_i &: \underbrace{6, 6, \dots, 6}_{50}, \underbrace{10, \dots, 10}_{30}, \underbrace{5, \dots, 5}_{20} \Rightarrow \bar{z} = 7
\end{aligned}$$

(16) a) $P(X_i = k) = P(X_1 = k) = P(\text{first } k \text{ non-aces, then an ace})$

$= P(\text{first } k \text{ are not aces})\, P((k+1)\text{-st is ace} \mid \text{first } k \text{ are not aces})$

$$= \frac{\binom{48}{k}}{\binom{52}{k}} \cdot \frac{4}{52-k}$$

b) Note that $X_1 + X_2 + X_3 + X_4 + X_5 = 48$ whatever the order of the cards.
Since each X_i has the same distribution, we get

$48 = E(X_1 + \dots + X_5) = E(X_1) + E(X_2) + \dots + E(X_5) = 5E(X_1) \Rightarrow E(X_1) = \frac{48}{5} = 9.6$

and then $E(X_i) = \frac{48}{5}$ for $i = 1, \dots, 5$

c) No: e.g. $P(X_1 = 48, X_2 = 1) = 0$ but $P(X_1 = 48)P(X_2 = 1) \neq 0$.

§3.2 (10) a) $I_A + I_B = \begin{cases} 0 & \text{if neither } A \text{ nor } B \text{ happens} \\ 1 & \text{if exactly one of } A \text{ and } B \text{ happens} \\ 2 & \text{if } A \text{ and } B \text{ both happen} \end{cases}$

Thus, $P(I_A + I_B = 0) = P(A^c B^c) = P(A^c)\,P(B^c) = (1-P(A))(1-P(B))$

$P(I_A + I_B = 1) = P(AB^c) + P(A^c B) = P(A)\,P(B^c) + P(A^c)\,P(B)$

$= P(A)(1-P(B)) + P(B)(1-P(A)) = P(A) + P(B) - 2P(AB)$

$P(I_A + I_B = 2) = P(AB) = P(A)\,P(B)$.

Then with $Y = (I_A + I_B)^2$, Range $(Y) = \{0,1,4\}$ and

$P(Y=0) = 1 - P(A) - P(B) + P(A)P(B)$, $P(Y=1) = P(A) + P(B) - 2P(AB)$,

$P(Y=4) = P(A)\,P(B)$

b) Note that $I_A^2 = I_A$, $I_B^2 = I_B$ ~~and $I_A I_B$~~, so:

$E[I_A + I_B]^2 = E(I_A^2 + 2 I_A I_B + I_B^2) = E(I_A + 2 I_A I_B + I_B)$

$= E(I_A) + 2E(I_A I_B) + E(I_B) \overset{\text{(indep.)}}{=} E(I_A) + 2E(I_A)E(I_B) + E(I_B)$

$= P(A) + 2P(A)P(B) + P(B)$.

(Alternative: use part a)).

(18) $\mu = E(X) = aP(X=a) + bP(X=b)$ and $P(X=a) + P(X=b) = 1$, so

$\mu = aP(X=a) + b(1-P(X=a)) = b - (b-a)P(X=a)$

$\therefore\ P(X=a) = \dfrac{b-\mu}{b-a}$, $\quad P(X=b) = \dfrac{\mu-a}{b-a}$.