Answer Key, Hw. #7

§2.4 (2) Number of successes in 500 trials is binomial (n,p) with $n=500$, $p=0.02$ so let $\mu = np = 10$.

a) $P(1 \text{ success}) = P(1) \approx e^{-10}\frac{10^1}{1!} = 10e^{-10} \doteq .000454$

b) $P(2 \text{ or fewer}) = P(0)+P(1)+P(2) \approx e^{-10}\left(\frac{10^0}{0!}+\frac{10^1}{1!}+\frac{10^2}{2!}\right) = 61e^{-10} \doteq .00277$

c) $P(\text{more than } 3) = 1 - P(3 \text{ or fewer}) = 1 - [P(0)+P(1)+P(2)+P(3)]$

$\approx 1 - .00277 - e^{-10}\frac{10^3}{3!} \doteq .9897$

(9) If a prize is considered a success, then $p = .95$, which is close to 1. Thus, the Poisson approximation can not be used directly! Instead, consider "prize" a failure, and "no prize" a success. Then $p = .05$ and $n = 52$, and the number of successes is binomial (n,p). So let $\mu = np = 2.6$, then

$P(\text{more than 45 prizes}) = P(46 \text{ or more failures}) = P(6 \text{ or fewer successes})$

$\approx \sum_{k=0}^{6} e^{-\mu}\frac{\mu^k}{k!} = e^{-2.6}\sum_{k=0}^{6}\frac{(2.6)^k}{k!} \doteq .9828$

§2.5 (6) a) $P(\text{no count cards}) = \binom{36}{13}\Big/\binom{52}{13}$.

b) $\{\text{no } J, Q, K\} = \{\text{no count cards}\} \cup \{\text{at least one } A, \text{ no } J, Q, K\}$

The right side is a union of disjoint events, and therefore,

$P(\text{at least one } A, \text{ no } J, Q, K) = P(\text{no } J, Q, K) - P(\text{no count cards})$

$$= \frac{\binom{40}{13}}{\binom{52}{13}} - \frac{\binom{36}{13}}{\binom{52}{13}} = \frac{\binom{40}{13}-\binom{36}{13}}{\binom{52}{13}}$$

c) $P(\text{at most one kind of count card}) = 4 \times$ (answer to part b)).

§2.5 (9) a) The second sample is drawn if and only if the first sample contains exactly one bad item, which happens with probability

$$P(\text{exactly one bad in first sample}) = \frac{\binom{10}{1}\binom{40}{4}}{\binom{50}{5}}$$

Given that the second sample is drawn, it contains more than one bad item with probability

$$1 - \frac{\binom{9}{0}\binom{36}{10}}{\binom{45}{10}} - \frac{\binom{9}{1}\binom{36}{9}}{\binom{45}{10}}$$

since there are now 9 bad items left, and 36 good ones. Thus, by the multiplication rule, the required probability is

$$\frac{\binom{10}{1}\binom{40}{4}}{\binom{50}{5}} \cdot \left[1 - \frac{\binom{9}{0}\binom{36}{10}}{\binom{45}{10}} - \frac{\binom{9}{1}\binom{36}{9}}{\binom{45}{10}}\right].$$

b) $P(\text{lot accepted}) = P(\text{no bad items in } 1^{st} \text{ sample})$
$+ P(\text{one bad item in } 1^{st},\ 0 \text{ or } 1 \text{ bad items in } 2^{nd})$

$$= \frac{\binom{40}{5}}{\binom{50}{5}} + \frac{\binom{10}{1}\binom{40}{4}}{\binom{50}{5}} \cdot \left[\frac{\binom{9}{0}\binom{36}{10} + \binom{9}{1}\binom{36}{9}}{\binom{45}{10}}\right].$$

(12) c) There are $13 \times 12 = 156$ ways to choose the two ranks a and b. For each, the number of 5-card hands with 3 "a"s and 2 "b"s is $\binom{4}{3}\binom{4}{2}\binom{44}{0} = 4 \cdot 6 \cdot 1 = 24$. Since there are $\binom{52}{5}$ possible hands overall,

$$P(\text{full house}) = \frac{156 \cdot 24}{\binom{52}{5}} \approx .00144$$