Answer Key, Hw. #7

§2.2 (14) a) The number of boxes having 390 or more working devices has a binomial (400, .95) distribution. Set $n=400$, $p=.95$, so $\mu=np=380$ and $\sigma=\sqrt{npq}=\sqrt{19}\doteq 4.36$. Then

$$P(\geq 390 \text{ working in box}) \approx 1-\Phi\left(\frac{390-\frac{1}{2}-\mu}{\sigma}\right)=1-\Phi\left(\frac{389.5-380}{4.36}\right)$$
$$=1-\Phi(2.18)=1-.9854=.0146.$$

b) $P(\geq k \text{ work}) \approx 1-\Phi\left(\frac{k-\frac{1}{2}-\mu}{\sigma}\right)=1-\Phi\left(\frac{k-380.5}{4.36}\right)=\Phi\left(\frac{380.5-k}{4.36}\right)\geq .95$

$\Leftrightarrow \frac{380.5-k}{4.36}\geq 1.65 \Leftrightarrow k\leq 373.3$

So the largest integer(!) k that guarantees a 95% success rate is 373.

§2.4 (2) Number of successes in 500 trials is binomial (n,p) with $n=500$, $p=0.02$ so let $\mu=np=10$.

a) $P(1 \text{ success})=P(1)\approx e^{-10}\frac{10^1}{1!}=10e^{-10}\doteq .000454$

b) $P(2 \text{ or fewer})=P(0)+P(1)+P(2)\approx e^{-10}\left(\frac{10^0}{0!}+\frac{10^1}{1!}+\frac{10^2}{2!}\right)=61e^{-10}\doteq .00277$

c) $P(\text{more than } 3)=1-P(3 \text{ or fewer})=1-[P(0)+P(1)+P(2)+P(3)]$

$\approx 1-.00277-e^{-10}\frac{10^3}{3!}\doteq .9897$

(9) If a prize is considered a success, then $p=.95$, which is close to 1. Thus, the Poisson approximation can not be used directly! Instead, consider "prize" a failure, and "no prize" a success. Then $p=.05$ and $n=52$, and the number of successes is binomial (n,p). So let $\mu=np=2.6$, then

$P(\text{more than 45 prizes})=P(46 \text{ or more failures})=P(6 \text{ or fewer successes})$

$$\approx\sum_{k=0}^{6}e^{-\mu}\frac{\mu^k}{k!}=e^{-2.6}\sum_{k=0}^{6}\frac{(2.6)^k}{k!}\doteq .9828$$

Answer Key, Hw. #6

§2.2 (9) a) The number of passengers that show up has a $Bin(n,p)$ distribution with $n = 324$, $p = 0.9$. Thus $\mu = np = 291.6$ and $\sigma = \sqrt{npq} = 5.4$, and

$$P(\text{flight overbooked}) = P(\geq 301 \text{ show up}) \approx 1 - \Phi\left(\frac{300.5 - 291.6}{5.4}\right)$$
$$= 1 - \Phi(1.65) = 1 - .9505 = .0495, \text{ or } 4.95\%.$$

b) Note that 300 is 92.6% of 324, so we are interested in the probability that the <u>proportion</u> of successes in n trials is $\geq 92.6\%$ when the mean proportion is 90%. In view of the Square Root Law / Law of Large Numbers (pp. 100/101), this probability decreases as n increases. Hence, when people travel in groups the probability of overbooking increases, since there are fewer independent trials (see c) below).

c) Now $n = \frac{324}{2} = 162$, $p = 0.9$, $\mu = 145.8$, $\sigma = 3.82$

(Note that, compared to a), both μ and the critical value of k were divided by 2, but σ was only divided by $\sqrt{2}$. This illustrates the square root law.) Now:

$$P(\text{flight overbooked}) = P(\geq 151 \text{ \underline{pairs} show up}) \approx 1 - \Phi\left(\frac{150.5 - 145.8}{3.82}\right)$$
$$= 1 - \Phi(1.23) = 1 - .8907 = .1093, \text{ or } 10.93\%.$$

§2.2 (14) a) The number of boxes having 390 or more working devices has a binomial (400, .95) distribution. Set $n = 400$, $p = .95$, so $\mu = np = 380$ and $\sigma = \sqrt{npq} = \sqrt{19} \doteq 4.36$. Then

$$P(\geq 390 \text{ working in box}) \approx 1 - \Phi\left(\frac{390 - \frac{1}{2} - \mu}{\sigma}\right) = 1 - \Phi\left(\frac{389.5 - 380}{4.36}\right)$$
$$= 1 - \Phi(2.18) = 1 - .9854 = .0146.$$

b) $$P(\geq k \text{ work}) \approx 1 - \Phi\left(\frac{k - \frac{1}{2} - \mu}{\sigma}\right) = 1 - \Phi\left(\frac{k - 380.5}{4.36}\right) = \Phi\left(\frac{380.5 - k}{4.36}\right) \geq .95$$
$$\Leftrightarrow \frac{380.5 - k}{4.36} \geq 1.65 \Leftrightarrow k \leq 373.3$$

So the largest integer(!) k that guarantees a 95% success rate is 373.