Math 3000 Review for Exam 1 **February 2017** **P. Allaart**

1. a) (*10 pts.*) Construct a truth table for the statement

$$[p \Rightarrow (q \wedge \sim q)] \Leftrightarrow \sim p$$

b) (*2 pts.*) Based on your truth table, is the statement in (a) a tautology?

2. (*9 pts.*) Write the negation of the following statement such that no negation symbol ($\sim$) appears in front of quantifiers or logical connectives.

$$\forall \varepsilon > 0, \exists \delta > 0 \ni \forall x \text{ and } \forall y, [|x-y| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon]$$

3. (*3 pts. each*) Indicate whether each statement is true, false, or not a statement at all. (No explanations needed.)

a) 3 is prime and 7 is odd.

b) 123456789 is a very big number.

c) π is rational if and only if 17 is divisible by 4.

d) If $5 < 7$ implies that $8 > 10$, then $5/2$ is an integer.

4. (*4 pts. each*) Determine whether each statement about the real numbers is true or false. **Justify each answer carefully.**

a) $\exists x \ni [x^2 = -2 \Rightarrow x^2 = 5]$

b) $\exists x \ni \forall y, x + y = 0$

c) $\forall x, \exists y$ and $\exists z \ni xy = z^2 + 1$

5. (*3 pts. each*) Find the following unions and/or intersections:

a) $\displaystyle\bigcap_{n=1}^{\infty} \left[\frac{1}{n}, 2 - \frac{1}{n}\right]$

b) $\displaystyle\bigcup_{n \in \mathbb{Z}} (n, n + 1.001)$

c) $\displaystyle\bigcap_{x \in [1,2]} (x, x+5)$

d) $\displaystyle\bigcup_{x \in [1,2]} (x, x+5)$

6. (*3 pts. each*) For each set below, find its interior, boundary and closure:

a) $S = (1, 4)$

b) $S = \mathbb{R}$

c) $S = \{1, 2, 3\} \cup \{x \in \mathbb{R} \,\big|\, |x - 3| < 1\}$

d) $S = \left\{ \frac{1}{n^2} \middle| n \in \mathbb{N} \right\}$

7. Recall that $A \backslash B = \{x | x \in A \text{ and } x \notin B\}$.

a) (*3 pts.*) Write the negation of the condition of membership in $A \backslash B$ (the bit behind "$x|$"). In other words, complete the following equivalence:

$x \notin A \backslash B \qquad \Longleftrightarrow$

b) (*11 pts.*) Let A and B be subsets of $\mathbb{R}$. Prove that

$$A \backslash (A \backslash B) = A \cap B,$$

by proving that each side is a subset of the other. (No credit for just drawing a Venn diagram!!!)

8. a) (*3 pts.*) Write what it means, *by definition*, for x to be an interior point of a set S.

b) (*10 pts.*) Let S and T be subsets of $\mathbb{R}$. Prove that

$$\text{int}\,(S) \cup \text{int}\,(T) \subseteq \text{int}\,(S \cup T).$$

c) (*4 pts.*) Give an example to show that the statement in (b) becomes false if "$\subseteq$" is replaced with "$=$". Justify your answer!

9. **Extra credit!!** (*8 pts.*)
Let S and T be subsets of $\mathbb{R}$. Prove that

$$\text{bd}(S \cup T) \subseteq \text{bd}(S) \cup \text{bd}(T).$$

Show ALL of the details!