Math 3000, Homework assignment #12

1. Read Section 2.4
2. Do the following problems from section 3.1: 4,11,17,20,23b,24,25. Turn in 4,20,23b.
3. Prove by induction that the union of any finite collection of closed subsets of $\mathbb{R}$ is closed. (You may assume the union of **two** closed sets is closed - we already proved this.) (*Hint*: Let $P(n)$ be the statement "If $S_1, \ldots, S_n$ are closed subsets of $\mathbb{R}$, then $\bigcup_{i=1}^{n} S_i$ is closed.")
4. For which positive integer amounts n cents can you make exact postage using only 4c and 7c stamps? Prove it two ways: (a) by induction and (b) by strong induction.
5. Let $f(x) = \sin x$. Prove by induction that
$$f^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right)$$
for all $n \in \mathbb{N}$, where $f^{(n)}$ denotes the nth order derivative of f.
6. Find a formula for the nth derivative of $f(x) = \ln x$. Prove by induction.
7. Turn in problems 3, 4 and 5 above.