Math 2700 - Review for Exam 3 (April 2014) **P. Allaart**

SHOW ALL YOUR WORK! NO WORK=NO CREDIT!!

No Calculators Allowed! - But you shouldn't need any.

1. *(11 pts.)* Find the determinant of the matrix

$$\begin{bmatrix} 1 & 0 & 5 & 3 \\ 3 & 0 & 2 & 4 \\ -1 & 2 & 1 & 4 \\ 1 & 0 & -2 & 0 \end{bmatrix}$$

You may use any method, but make sure that each step can be clearly understood!

2. *(8 pts.)* Show that if A is an invertible matrix, then A^{17} is also invertible.

3. *(10 pts.)* Let A be a 4×4 square matrix with $\det A = 3$, and suppose the following elementary row operations transform A into a matrix B:
   1. Add two times row 1 to row 3.
   2. Interchange row 2 and row 4.
   3. Subtract three times row 2 from row 3.
   4. Multiply row 3 by 2.
   5. Multiply row 4 by -1.

Find $\det B$. Explain your reasoning!

4. Let A be an 8×5 matrix.

   a) *(6 pts.)* Could the rank of A possibly be 6? Explain *briefly*.

   b) *(6 pts.)* If you knew that $\dim \operatorname{Nul} A = 1$, how many rows of all zeros would an echelon form of A have? Explain *briefly*.

5. Let

$$A = \begin{bmatrix} -1 & 3 & 1 & -2 & -5 \\ 0 & 1 & 1 & 0 & 2 \\ 4 & -7 & 1 & 6 & 0 \end{bmatrix}$$

   a) *(7 pts.)* Find a basis for $\operatorname{Nul} A$, and give the dimension of $\operatorname{Nul} A$.

   b) *(7 pts.)* Find a basis for $\operatorname{Col} A$, and give the dimension of $\operatorname{Col} A$.

6. *(6 pts. each)* Determine whether each set is a subspace of $\mathbb{R}^3$. You may use a theorem from the book, but your argument must be clear and complete.

   a) $H = \left\{ \begin{bmatrix} s + 2t \\ -t \\ -4s + 3t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\}$

   b) $K = \left\{ \begin{bmatrix} a + b \\ a - 3 \\ 2a - 5b \end{bmatrix} : a, b \text{ in } \mathbb{R} \right\}$

7. *(10 pts.)* Find a basis for the set of all vectors of the form

$$\begin{bmatrix} a - 2b + 5c \\ 2a + 5b - 8c \\ -a - 4b + 7c \\ 3a + b + c \end{bmatrix}$$

(Be careful!)

8. *(10 pts.)* Let H be the set of all 3×3 symmetric matrices (i.e. matrices A such that $A^T = A$). Show that H is a subspace of $M_{3\times 3}$.

9. Let $T : \mathbb{P}_2 \to \mathbb{P}_1$ be the mapping $T(\mathbf{p}) = \mathbf{p}(0) + \mathbf{p}(1)t$.

a) *(5 pts.)* Show that T is a linear transformation.

b) *(5 pts.)* Find a basis for the kernel of T.

c) *(5 pts.)* Prove that the range of T is all of $\mathbb{P}_1$.

(Hint: a typical element of $\mathbb{P}_1$ is of the form $a + bt$. Construct a polynomial $\mathbf{p}(t)$ in $\mathbb{P}_2$ such that $\mathbf{p}(0) = a$ and $\mathbf{p}(1) = b$.)

10. Let $A = \begin{bmatrix} 2 & 1 \\ 7 & -4 \end{bmatrix}$. Find the eigenvalues of A, and find at least one eigenvector for each eigenvalue.

11. Given that 4 is an eigenvalue of the matrix

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -2 & 10 & -4 \\ -1 & 3 & 2 \end{bmatrix},$$

find a basis for the corresponding eigenspace.

12. **Extra credit!!** Given subspaces H and K of a vector space V, the **sum** of H and K, written $H + K$, is the set of all vectors that can be written as the sum of one vector in H and one vector in K. That is,

$$H + K = \{\mathbf{w} : \mathbf{w} = \mathbf{u} + \mathbf{v} \text{ for some } \mathbf{u} \text{ in } H \text{ and some } \mathbf{v} \text{ in } K\}$$

a) *(5 pts.)* Show that $H + K$ is a subspace of V.

b) *(5 pts.)* Determine whether H a subspace of $H + K$.